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1. REPORT DATE (DD-MM-YYYY) 25-01-2012		2. REPORT TYPE Final Performance			3. DATES COVERED (From - To) 01 February 2008 to 31 May 2011	
4. TITLE AND SUBTITLE An Analytical Framework for Fast Estimation of Capacity and Performance in Communication Networks				5a. CONTRACT NUMBER		
				5b. GRANT NUMBER FA9550-08-1-0064		
				5c. PROGRAM ELEMENT NUMBER		
6. AUTHOR(S) Iraj Saniee, Yuliy Baryshnikov (Bell Labs Research, Alcatel-Lucent), Onuttom Narayan (Dept of Physics, University of California, Santa Cruz)				5d. PROJECT NUMBER		
				5e. TASK NUMBER		
				5f. WORK UNIT NUMBER		
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Bell Labs Research, Alcatel-Lucent Dept of Physics, University of California, Santa Cruz					8. PERFORMING ORGANIZATION REPORT NUMBER	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) AFOSR 875 N. Randolph St. Suite 325 Arlington, VA 22203					10. SPONSOR/MONITOR'S ACRONYM(S)	
					11. SPONSOR/MONITOR'S REPORT NUMBER(S) AFRL-OSR-VA-TR-2012-0212	
12. DISTRIBUTION/AVAILABILITY STATEMENT Unlimited						
13. SUPPLEMENTARY NOTES						
14. ABSTRACT The primary goal of this project was to create analytical methods for estimating such metrics as performance/congestion and capacity in large-scale communication networks. Since real networks exhibit a dizzying variety of forms and features, focus was placed on understanding fundamental intrinsic geometric and topological features with the hope that the said metrics could be derived parsimoniously from global features rather than through complex local features which would have made the goal of this project unachievable.						
15. SUBJECT TERMS						
16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT UNL	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON Iraj Saniee	
a. REPORT U	b. ABSTRACT U	c. THIS PAGE U			19b. TELEPHONE NUMBER (Include area code)	

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Final Performance Report

2008-2011

Grant Title: An Analytical Framework for Fast Estimation of Capacity and Performance in Communication Networks

Grant Award Number: FA9550-08-1-0064

Principal Investigators: Iraj Saniee, Yuliy Baryshnikov (Bell Labs Research, Alcatel-Lucent), Onuttom Narayan (Dept of Physics, University of California, Santa Cruz)

Reporting Period: 01 February 2008 to 31 May 2011

Project Accomplishments

The primary goal of this project was to create analytical methods for estimating such metrics as performance/congestion and capacity in large-scale communication networks. Since real networks exhibit a dizzying variety of forms and features, focus was placed on understanding fundamental intrinsic geometric and topological features with the hope that the said metrics could be derived parsimoniously from global features rather than through complex local features which would have made the goal of this project unachievable.

Up to 2007, there had been little research that attempted to cast the study of large-scale real-life networks as a branch of geometry. We therefore took our starting position from the work of Gromov and his geometric notion of large-scale (negative) curvature of geodesic metric spaces (GMS), since any graph with the natural or reasonable link-metric would be an instance of a GMS. Indeed, the “small world” property observed ubiquitously in man-made and natural networks hinted at the possibility that these structures would likely have “tree-like” geometry and thus exhibit some form of negative curvature. Negative curvature in the large, in the sense of Gromov, could thus provide a formalism to quantify the degree to which large-scale networks could prove to be tree-like, as quantified through the thin triangle condition and their universal δ -hyperbolicity.

Our first report [1] leveraged existing connectivity data at the IP-layer of a dozen global networks as measured by the RocketFuel team (funded by NSF in the early to mid 2000s). In our work, we created an experimental formalism – the curvature plot – which helped quantify “ δ -hyperbolicity” of large-scale networks and compared and contrasted these with the hyperbolicity (or lack thereof) of a dozen or so well-understood networks, from regular Euclidean and hyperbolic lattices to various random graphs. The results provided strong

evidence of the possibility of hyperbolicity of these networks as well as a subset of better-understood classical networks.

A key observation from our work was the notion that δ -hyperbolic networks would likely have an inherent non-empty core through which a fixed fraction of the total load of the network passes. Thus for a network of size N we expect $O(N^2)$ scaling of load in the network core. We also provided a proof of $O(N^2)$ scaling of the core load in power-law networks which satisfy some general scaling assumptions, and showed that these assumptions apply to preferential attachment models [2]. As a byproduct, we showed that the probability of a node having a load l scales as $1/l^2$ in such models, a point of controversy prior to our work. A formal proof of $O(N^2)$ scaling of load in δ -hyperbolic networks was subsequently furnished in [3,4,5].

Another key observation from the curvature plots in [1] was that the standard random graph (due to Erdos-Renyi) in the regime where the average degrees remain fixed (and above 1) and the number of nodes get large, is not hyperbolic. This was surprising since it was generally assumed that random graphs are locally tree-like and in addition have a positive spectral gap. These two hint strongly at the possibility of hyperbolicity. In [6] we provided a formal proof that E-R random graphs are neither hyperbolic nor do they have a positive spectral gap. We elucidated that the anecdotal spectral gap in random graphs arises in the regime in which degrees scale like $\log(N)$ and are not constant. We also elucidated that the notion of linear isoperimetry in hyperbolic geometry is different from that used in the graph theory literature, another source of confusion since some δ -hyperbolic networks may indeed have a zero spectral gap even though they satisfy a linear (hyperbolic) isoperimetric inequality and not a linear (graph theoretic) isoperimetric inequality.

The latter observation was a starting point to better understand why the Cheeger ratio in finite regular trees differed from that of the infinite regular tree. Using Dirichlet eigenvalues in place of standard eigenvalues, we provided a proof in [7] of convergence of this ratio for finite graphs as the size scales up to infinity. This machinery also enabled us to use a new way to use spectral clustering for detection of communities (sub-graphs with small cuts) and demonstrated this by showing novel clusterings of the dozen RocketFuel networks, our starting point in this effort.

In conclusion, the notion of curvature plot we introduced in [1] is likely to become a basic tool for the analysis of large-scale networks and specifically for detection of negative curvature in the large. As such, we have started exploring computationally simpler approaches for identification of hyperbolicity via the 4-point condition and are also expanding the underlying sampling method to explore more adequately the tails of various distributions that arise in the quantification of curvature. These form the basis of our ongoing work. This

grant enabled us to study and provide new tools based on geometry and topology for estimation of congestion and capacity in large-scale complex networks with much potential for applications beyond communication networks, to biological and social networks where much local measurements currently dominate alternative approaches that would seek fundamental insights gained from geometry.

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